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ABSTRACT

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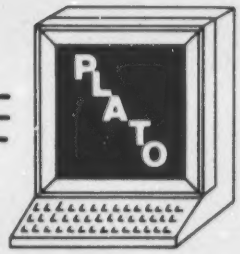

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A MODEL FOR INCORPORATING RESPONSE-TIME DATA IN SCORING ACHIEVEMENT TESTS

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by

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and
Maurice Tatsuoka

University of Illinois at Urbana-Champaign

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The differences in type of information-processing skill developed by different instructional backgrounds affect, negatively or positively, the learning of further advanced instructional materials. That is, if prior and subsequent instructional methods are different, a proactive inhibition effect produces low achievement scores on a posttest. This fact poses a serious problem for routing of students to an instructional level on the sole basis of performance on a diagnostic adaptive test.

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It is essential that we somehow unravel what information-processing strategy was used and consider this knowledge simultaneously.

Fortunately, response time often provides supplementary information which differentiates among individuals showing identical quality of performance. A model that reflects this kind of information, obtainable from response time scores, is formulated in a similar manner to latent trait theory and is discussed. This model is useful in identifying discriminating items that are sensitive to differences in instructional method. It also is helpful in identifying an individual's instructional background to a certain extent.

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ABSTRACT

The differences in type of information-processing skill developed by different instructional backgrounds affect, negatively or positively, the learning of further advanced instructional materials. That is, if prior and subsequent instructional methods are different, a proactive inhibition effect produces low achievement scores on a posttest. This fact poses a serious problem for routing of students to an instructional level on the sole basis of performance on a diagnostic adaptive test. It is essential that we somehow unravel what information-processing strategy was used and consider this knowledge simultaneously.

Fortunately, response time often provides supplementary information which differentiates among individuals showing identical quality of performance. A model that reflects this kind of information, obtainable from response time scores, is formulated in a similar manner to latent trait theory and is discussed. This model is useful in identifying discriminating items that are sensitive to differences in instructional method. It also is helpful in identifying an individual's instructional background to a certain extent.

INTRODUCTION

The study by Tatsuoka and Birenbaum (1979) raised an important issue with respect to adaptive diagnostic testing and computer-managed routing by which each examinee is sent to his/her level of instruction. It is necessary to consider an alternative scoring procedure in which individual differences in information-processing skills have been taken into account along with individual ability or achievement levels.

In Tatsuoka and Birenbaum's study, a computerized diagnostic adaptive test for a series of pre-algebra signed-number lessons was given to eighth-graders at a junior high school, and a computer-managed routing system sent each examinee to the instructional unit corresponding to the level of skill at which he/she ended up in the initial test. The adaptive test for signed-numbers consisted of 12 groups of items representing 12 different skills. The instructional units of computerized lessons that teach the same 12 skills were rearranged into the same order as the skills in the adaptive test, so that if an examinee stopped at the seventh skill level, he/she was sent to the seventh level of the lessons. After this student went through the 7th to 12th instructional units, a 52-item, conventional computerized posttest was administered.

Factor analysis revealed that the test scores of the posttest were far from satisfying the assumption of local independence--i.e., unidimensionality. A further, close investigation was done by performing a cluster analysis on the 92 examinees' response patterns on the basis of Euclidean distances between pairs of response vectors. The result of this analysis led us to find a group of students whose response patterns were

significantly different from others. Their scores on the items prior to the stopping level of the initial diagnostic test were higher than most scores of other students, while their scores on the the subsequent items were as low as the poorest students' scores. We confirmed with their teachers that most of them were actually A-students. We also confirmed that the members of this group were taught signed-number addition operations by a teaching method different from that of subsequent instructional units which teach subtraction operations. The procedure of information processing associated with these two instructional methods of performing arithmetic upon signed numbers are greatly different. The traditional scoring procedure of the latent trait theory would not be capable of detecting these discrepancies associated with different information processes for arriving at the answers to a given item.

A study by Tatsuoka and Tatsuoka (1978) indicates one useful approach toward the goal just mentioned. They showed that under certain general conditions, item response time scores very closely follow Weibull distributions--a three parameter family extensively used in system reliability theory (see, e.g., Mann et al., 1974). The most interesting of the three parameters is the shape parameter, whose magnitude determines the nature of the "conditional response rate," that is, the conditional probability that an examinee who has not responded to an item up to time t will respond to it within an infinitesimally short time interval thereafter. A brief note on the mathematical and conceptual backgrounds of the Weibull distribution, introduced in our previous study, will be described in the following section.

As a follow-up to the Tatsuoka-Birenbaum study, Weibull distributions were fitted to every item in the posttest. The Weibull fit of

almost all items--14 items on addition which were taught prior to the students' exposure to the PLATO lessons--was quite poor when the fitting was done for the total sample. However, the separate fits in two groups that had been earlier identified as having distinctly different instructional backgrounds were very good for all 14 items (see Appendix 1-a, 1-b, and 1-c). Further, it was found that the value of shape parameter c differed considerably in the two groups for each item, being higher in one group for some items and lower for others. That is, there was a task by instructional method interaction effect on the shape parameter c.

The foregoing suggests that the Weibull shape parameter can give us a handle on identifying items that are sensitive to particular information processing skills. After identifying and constructing such discriminating items, it was anticipated that an index known as "person conditional response rate" (PCRR), to be developed below, could be used for "postdicting" the instructional background of students and routing them accordingly.

RATIONALE OF WEIBULL DISTRIBUTIONS

Measuring the time needed to achieve a given goal (that is, response time) is easy when we use computer-managed testing, but since it is impossible to collect accurate time data in paper-and-pencil testing, the latter has never been utilized so far in the realm of practical application of psychometrics. Tatsuoka and Tatsuoka (1978) have studied the statistical aspects of response time distributions and their characteristics associated with test-items.

There are a number of theoretical distributions by which the response time data may seem to be fitted well, so it is necessary to follow

some guidelines as to what sort of distribution might be appropriate to represent a set of response times for a given item. Rasch (1960) used the two-parameter gamma distribution as a model for the time taken to read a passage of N words. He used the Poisson process as a guide to his model. The occurrence of a response is a random event, and all the random events are assumed to be of the same kind. He was interested in their total number.

Sato and others (1975) introduced the Weibull distribution, which has been used extensively in the context of system-reliability theory. Reliability theory is the study of the probability of failure, within a given time span, of a mechanical or electronic system as a function of the probabilities of failure of individual components of the system. Their rationale for diverting a distribution from such an alien field is that the test item is identified with the system whose "longevity" is being assessed; the student's "attacks on the item" correspond to the shocks or wear and tear to which the system is subjected, and the eventual solution of the item is the failure of the system. It is plausible to imagine the student as being intent on "cracking the system" by answering the item correctly. The time he/she takes in doing so--the response time--corresponds to the "survival time" of the system. This rationale for the applicability of Weibull distributions for item response time does not lead to a derivation of the distribution or density function. Mann and others (1974) have said the distribution was empirically discovered rather than deductively derived in the first place. But later, a logical basis was postulated as an ex post facto rationalization, and it added greatly to the credibility of the distribution in the theory of system reliability. This

is the concept of hazard rates, which is essentially the conditional probability that a system which has survived through time t will fail during an infinitesimal time interval immediately after that.

We introduced a similar concept "conditional response rate" (CRR) in our study as a logical basis. Suppose f(t) is the probability density that a person randomly selected from the population will respond to a given item during the interval [t, t+dt]. Then, the proportion of individuals who will have responded to the item by time t is the probability distribution function $F(t) = \int_{to}^{t} f(u)du$. The proportion of individuals have not responded to the item by time t is 1 - F(t). Consequently, the conditional probability density that a person will respond to the item during the interval [t, t+dt] given that he or she has not responded to the item up to time t is given by f(t)/[1-F(t)].

By assuming CRR as a function of time t to be monotonically increasing, or decreasing, as a power function of t, the Weibull distribution and density functions can be exprssed as follows:

$$(1) \qquad F(t) = \begin{cases} 1 - \exp\left[-\left(\frac{t-t_0}{u}\right)^c\right] \\ 0 \end{cases}$$

$$(2) \qquad f(t) = \begin{cases} \frac{c}{u}\left(\frac{t-t_0}{u}\right)^{c-1} \exp\left[-\left(\frac{t-t_0}{u}\right)^c\right] \\ 0 \end{cases}$$

where c(>0) is the shape parameter, u(>0) is the scale parameter, and t_0(>0) is the location parameter.

If $\underline{c} = 1$, then f(t) is a negative exponential density function. If $\underline{c}$ is less than 1, then f(t) is monotonically decreasing function. The Weibull density function is symmetric when $\underline{c}$ is about 3.6. Figure 2 is a

$$f(t) = (c/\mu_\emptyset{}^c)\,(t-t_\emptyset)^{c-1} * \exp\left[-\left((t-t_\emptyset)/\mu_\emptyset\right)^c\right]$$

c	$\mu_\emptyset$	$t_\emptyset$
.7	15	2
1	15	2
1.5	15	2
2.5	15	2

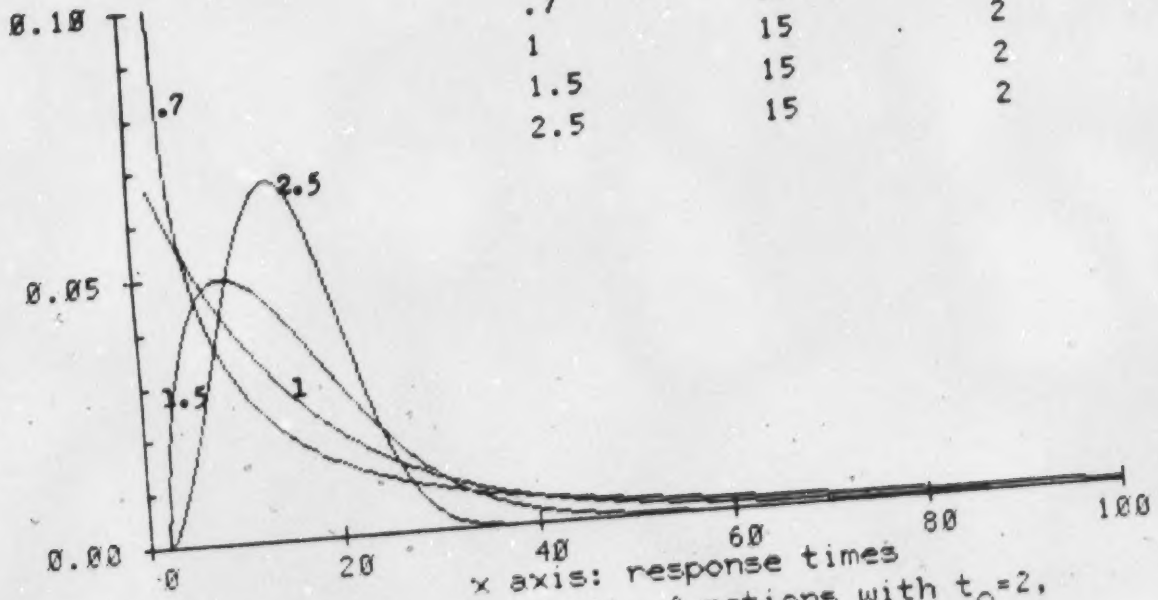
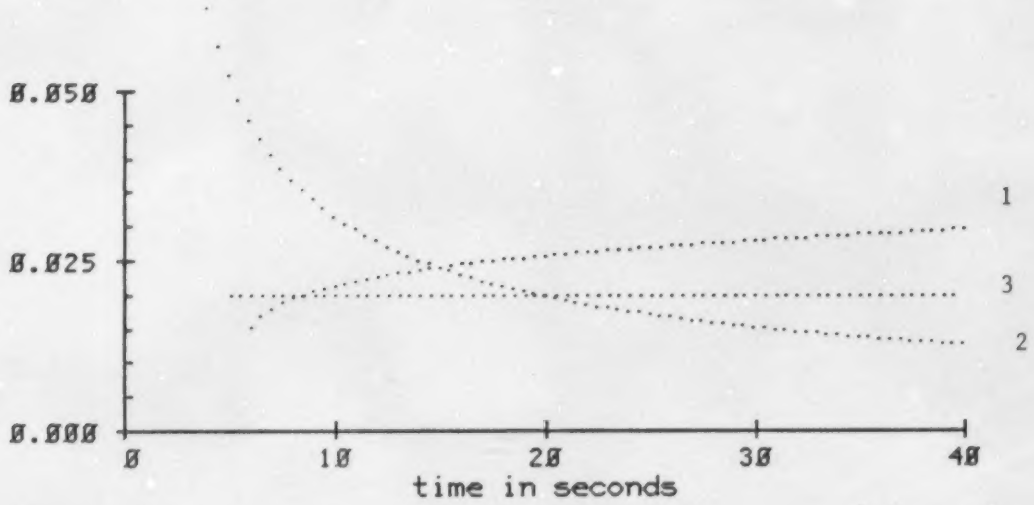


Figure 1. Weibull density functions with $t_\emptyset = 2$, $\mu_\emptyset = 15$, and four values of c



	$t_Ø$	c	$\mu_Ø$
1	5.47	1.16	38.47
2	.92	.38	19.33
3	5.ØØ	1.ØØ	5Ø.ØØ

Figure 2 Three types of Conditional Response Rate Function

copy of the conditional response rate function obtained from live data. The increasing curve is the CRR when c is larger than 1, and the decreasing dot-graph can be obtained from the distribution when c is less than 1. When c = 1, CRR becomes a straight line which is parallel to the time axis.

Figures 3 and 4 show the displays of "goodness of fit" tests with the normal and Weibull distributions. The step function represents the cumulative distribution of a set of response times to an item which asks about matrix multiplication. The continuous line stands for the estimated theoretical distribution function. The Weibull distribution fits the data better than does the normal distribution. About 700 cases of the "goodness of fit" test were carried out and most data fitted either the Weibull or the three parameter gamma distributions.

Theoretical distributions were fitted to the observed response time distribution of each item in two ways: first for the subgroup of students who answered the item correctly (OK subgroup), and second for the subgroup of students who answered the item incorrectly (NO subgroup). The OK subgroup and NO subgroup had considerably different estimated Weibull parameters but both showed very good fits for most items. Figure 5 shows the estimated Weibull distributions of the OK subgroup and NO subgroup for an item in the pretest which required matrix multiplication.

The Weibull parameter c of the OK subgroup in a 48-item matrix algebra pretest correlates with the numbers of options in the item (.32) and the difficulty indices (.41). The items with more choice options tended to have large c-values. If our interpretation, made in Tatsuoka & Tatsuoka (1978), that the item c-value reflects the degree of engagement

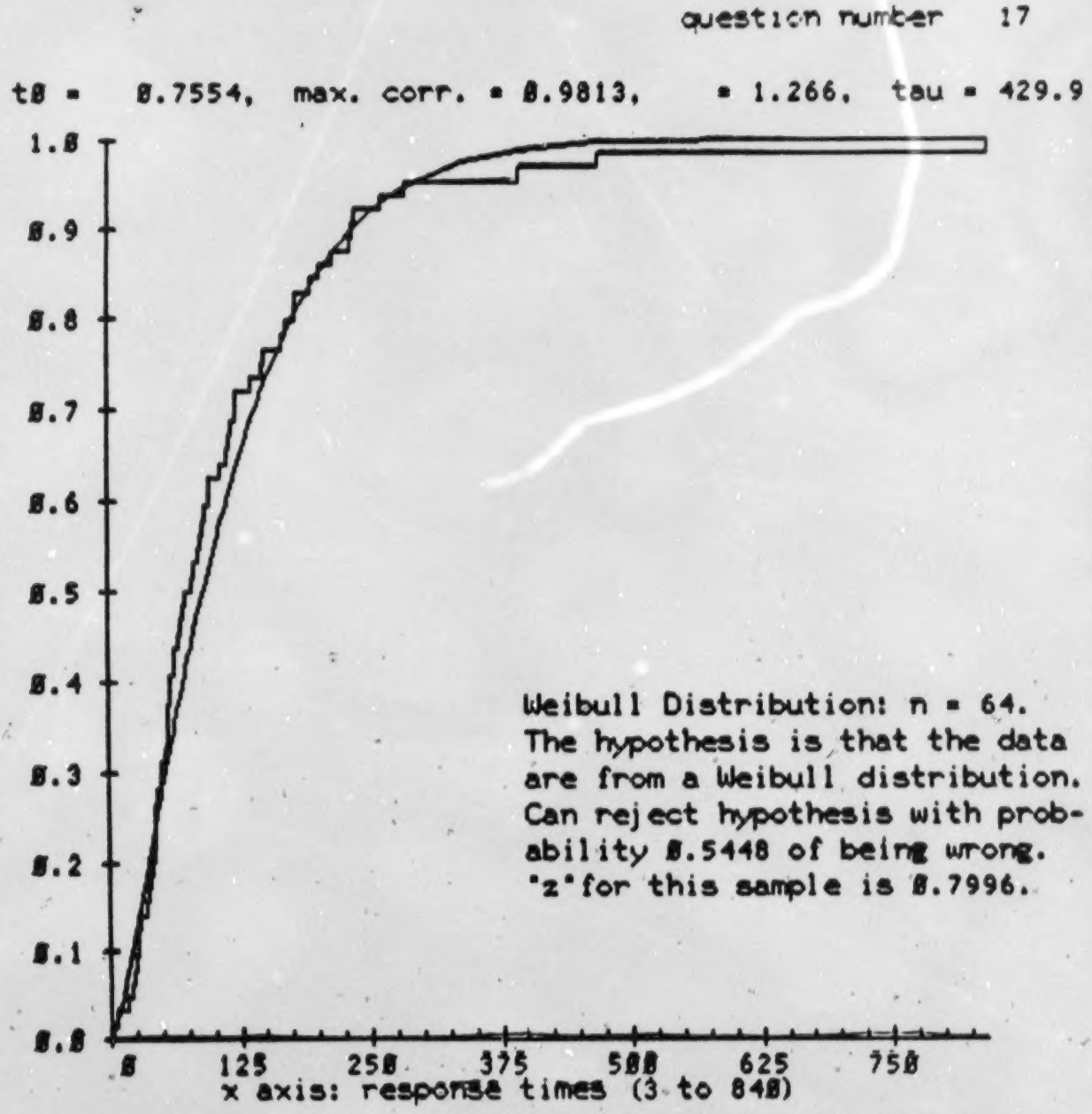


Figure 3 Goodness of fit test for the time data and weibull distribution function

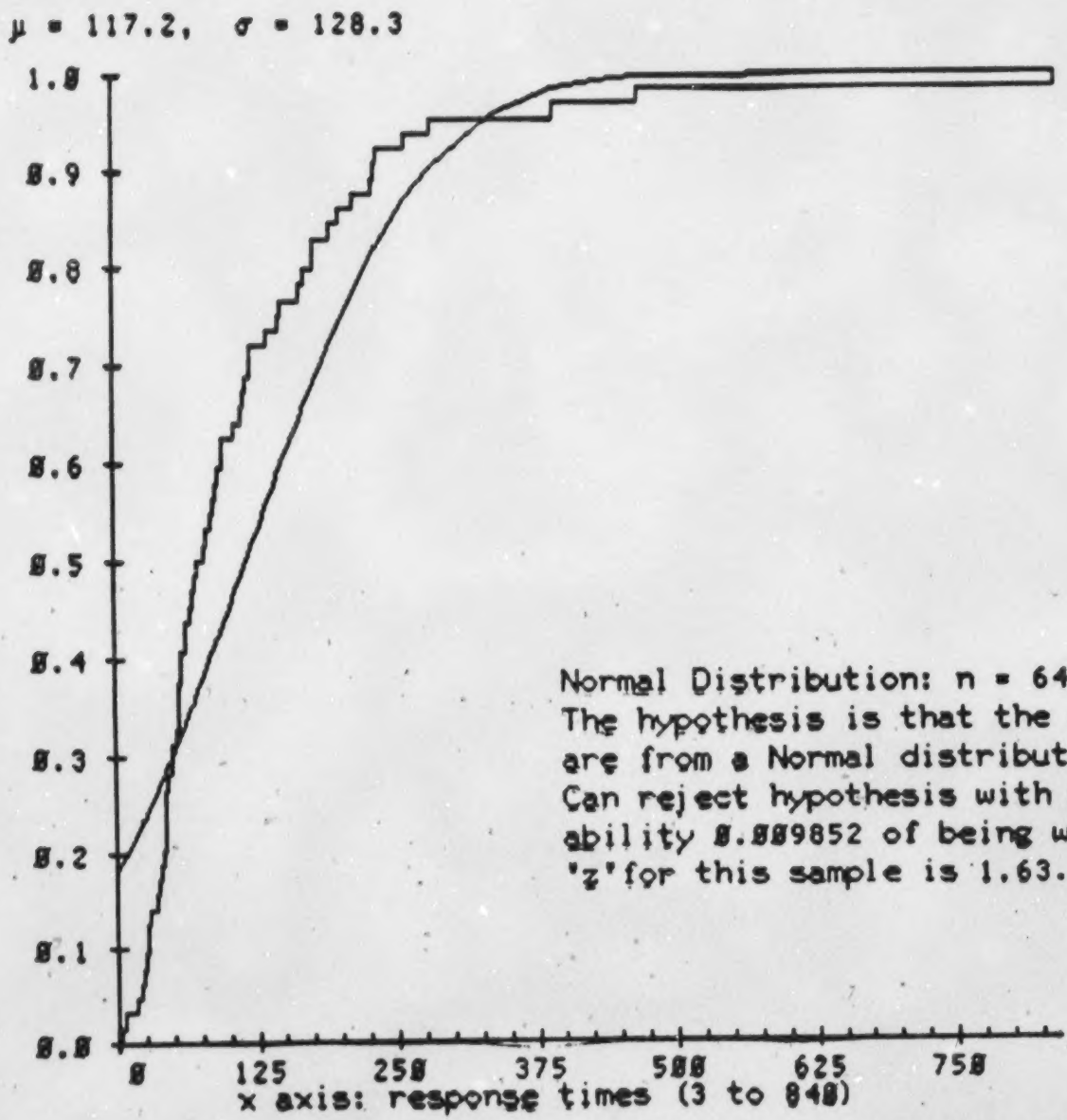


Figure 4 Goodness of fit test for the response time data and normal distribution

Weibull Distribution and Density

question number 16

	c	t_0	μ_0	mean
1 = okonly	1.161	5.4740	38.47	36.51
2 = noonly	0.3764	0.9206	19.33	76.68

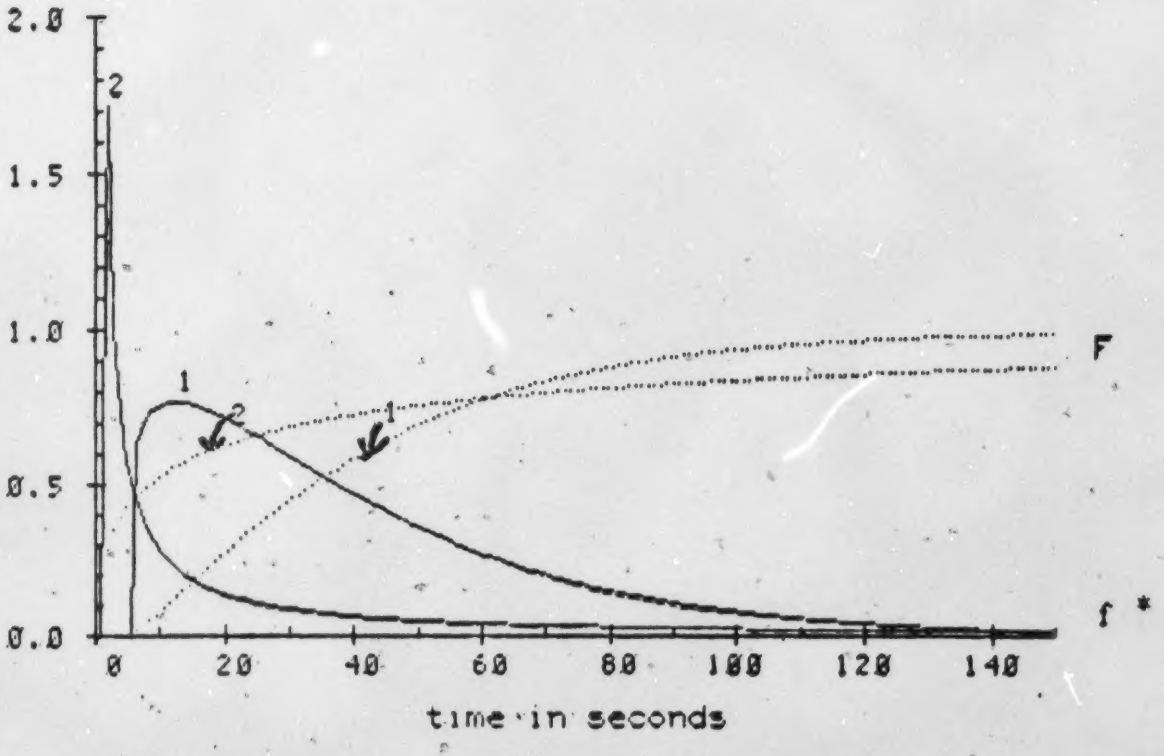


Figure 5 Weibull distribution and density functions

*Vertical scale for f(t) is magnified by a factor of u_0

students show when the item is correct, we may conclude that within the range represented, the larger the number of options, the greater the engagement students feel. This seems reasonable since items with more options present more of a cognitive task and hence probably induce greater involvement on the part of students. About ten items in the test asking mathematical properties of orthogonal transformations, eigen-values and -vectors, were very difficult for many students in the course. These items tended to have the smaller Weibull shape parameters c in both OK and NO subgroups. A similar observation was obtained from the 64-item signed-number pretest.

The three parameter gamma distributions fit well the items which repeatedly require a simple mechanical task, while the Weibull distributions fit well the items which require a higher cognitive task to respond to. Since the CRR of the gamma distributions is always non-decreasing, that is, either monotonically increasing or parallel to the time axis (see Appendix 2), the interpretation of the Weibull shape parameter (see Figure 2) provides wider applicabilty than the gamma shape parameter does. Moreover, the parameter estimation routine by maximum likelihood usually fails to give us convergent estimated gamma parameters when items have decreasing CRRs.

LATENT RESPONSE TIME MODEL

Latent Response Time Variable and Item Response Time Characteristic Curve

As a first step toward developing the person conditional response rate (PCRR), we postulate the existence of a latent response time variable,

(τ), analogous to the ability variable Ω in latent trait theory. Thus, given a set of n items, the performance on which is affected by θ, we assume that there also exists a variable which affects the time taken by an examinee to answer each of these items. We shall not attempt to give any precise psychological meaning to this construct beyond saying that it may be regarded as a pervasive trait of individuals to be slow or quick in solving items in a certain domain.

The plausibility of this postulation is suggested by the following empirical findings. In the Tatsuoka and Tatsuoka study (1978), the performance scores on a 48-item matrix algebra test were found to have a strong tendency toward unidimensionality. At the same time, the response times for these items showed a suggestion of unidimensionality by the scree test. On the other hand, the posttest for the signed-number lessons mentioned earlier showed no semblance of unidimensionality in the total sample. However, when one instructional-background group (we will call this Group 2 hereafter) identified by cluster analysis was removed, both performance scores and response times came somewhat closer to being unidimensional in the remaining sample.

On the strength of these observations and of the fact, mentioned earlier, that the Weibull distribution fits the response-time data for most items, we develop a model for item response time in the following manner, roughly paralleling latent trait theory.

Let

$$(3) \quad d_{ig} = t_{ig} - \bar{t}_{i}.$$

be the deviation of individual i's response time t_{ig} for item g from his or her mean response time $\bar{t}_{i.}$ over the given set of $\underline{n}$ items. Now τ be conceptualized as the expectation $E(t_{ig})$ of that person's response times over an infinite number of items of the same type as those in our set of $\underline{n}$. Then $\tau_i + d_{ig}$ is approximately equal to t_{ig}, so if we let i vary across the population, it is reasonable to assume that $\tau_i + d_{ig}$ follows a Weibull distribution just as t_{ig} does. We therefore define

$$(4) \qquad F_{d_g}(\tau) = 1 - \exp\left[-\left(\frac{\tau + d_g}{u_g}\right)^{c_g}\right]$$

as the "response-time characteristic function" (RTCF) for item g, where we have taken $t_o = 0$ in the general expression (2) for the Weibull distribution function to simplify the task of parameter estimation. This is interpreted to represent the probability that a person whose latent response time is τ will arrive at the answer to item g at or prior to time $\tau + d_g$.

For estimating the two item parameters c_g and u_g as well as the person parameter τ_i, the density function corresponding to (4) is written in accordance with Equation (1) (with t_o set equal to 0) for each person i, and the product over all items and those individuals who got the item correct is formed to obtain the likelihood function. That is,

$$(5) \quad L = \prod_{g=1}^{n} \prod_{i \in N_g} f(\tau_i + d_{ig}) = \prod_{g=1}^{n} \prod_{i=1}^{N_g} \exp\left[-\left(\frac{\tau_i + d_{ig}}{u_g}\right)^{c_g}\right] \frac{c_g}{u_g} \left(\frac{\tau_i + d_{ig}}{u_g}\right)^{c_g - 1}$$

where N_g is the number of subjects in OK-Subgroup for item g.

Before going to the next step of developing the PCRR function $G(\tau)$, let us see how τ itself, once estimated, can help in the task of

postdicting a student's instructional background. Suppose there are two items that differentiate between two prior instructions A and B by actually showing a reversal in the magnitude-order of mean times required for their solution by examinees who were previously taught by these two methods. Table 1 shows the mean response time (also with the estimates of Weibull parameter $\underline{c}$ and CRR) of 12 items described earlier for the two groups, of which prior instructional methods are A and B respectively. Let us take the means of items 32 and 33 as an example, then

$$t_{1A} = 9.65 \text{ sec.}, \quad t_{1B} = 5.43 \text{ sec.}$$

$$t_{2A} = 4.62 \text{ sec.}, \quad t_{2B} = 7.22 \text{ sec.}$$

Given these data and the observed response times t_1 and t_2 for the two items of a person about whom we have no other information, a natural but simple-minded decision rule for postdicting his/her instructional background would be to choose A if $t_1 > t_2$ and B, otherwise. The trouble, of course, is that the magnitude-order of the two observed times could be reversed from the "true" order by errors of measurement. Knowledge of the person's τ_1 may help increase our confidence in our postdiction, using the following sequential decision rule--again deliberately a simple-minded one.

We first administer only item 1 to this person. Now suppose his/her τ_1 is less than 6.73 sec. (the mean of the four mean response time listed above). Then, if $t_1 \geq 9.65$, we choose A, and the testing is terminated. If, on the other hand, $t_1 < 9.65$, we further administer item 2, and choose B iff $t_2 \geq 7.22$, and otherwise choose A or B according as $t_1 > t_2$ or $t_1 < t_2$, respectively. When the person's τ_1 is greater than or equal to 6.73, the sequential decision will be the dual of the above.

Table 1

The Means of Response Time, Observed Conditional Response Rate at Mean and Weibull Shape Parameters of Addition Problems of 64-item Signed Number Test

Item	Mean		CRR at Mean		Shape Parameter	
	Others	Group 2	Others	Group 2	Others	Group 2
3	9.84	13.14	0.13	0.07	1.45	.80
4	7.61	5.13	0.13	0.20	0.99	1.01
14	14.99	18.48	0.10	0.08	1.78	1.68
17	6.39	8.60	0.19	0.11	1.35	0.86
18	8.35	9.00	0.17	0.10	1.68	0.90
19	7.16	8.44	0.18	0.10	1.47	0.75
28	11.78	11.89	0.10	0.13	1.25	1.96
31	8.70	10.85	0.12	0.09	1.00	0.91
32	9.65	5.43	0.09	0.23	0.84	1.43
33	4.62	7.22	0.28	0.17	1.49	1.46
42	14.28	14.85	0.10	0.07	1.76	1.16
46	10.08	9.83	0.10	0.10	0.93	1.03
47	6.22	11.55	0.17	0.07	1.05	0.63
56	10.56	9.69	0.14	0.14	1.87	1.64

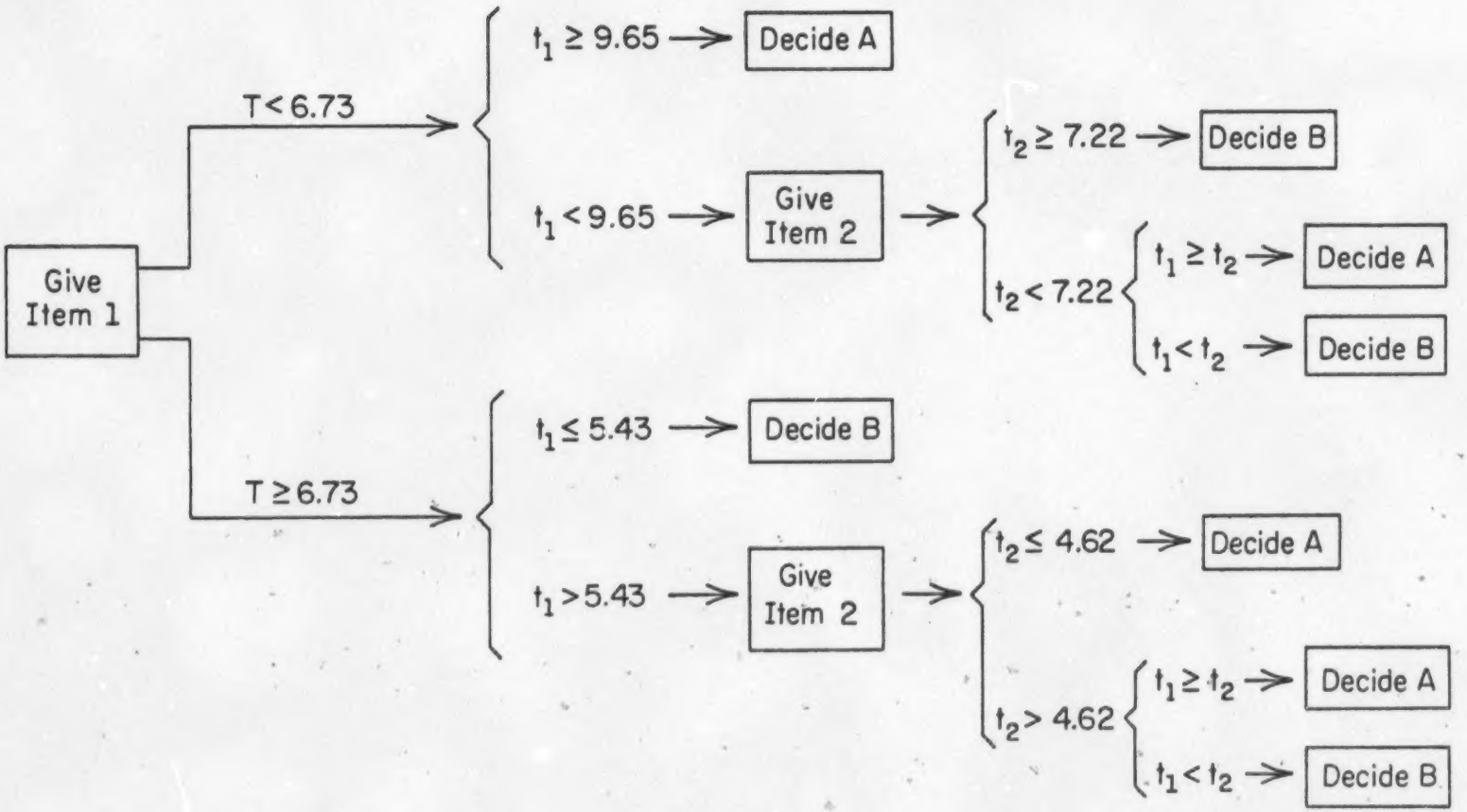


FIGURE 6 Sequential decision for Postdicting Method A or B based on knowledge of T and Response Times for Items 1 and 2.

Namely, if $t_1 \leq 5.43$, we choose B; if $t_1 > 5.43$ we further administer item 2, and choose A iff $t_2 \leq 4.62$. If $t_2 > 4.62$ we choose A or B according to the magnitude-order of t_1 and t_2.

The above discussion was kept at a simple, intuitive level to highlight the meaning of and how it would affect our interpretation of an observed response time. Refinements would include getting conditional response-rate distributions for each item given instruction background and the value of τ. Further, with a suitable assumption concerning the distribution of τ, one could derive the posterior probability for each instructional background given τ and the magnitude for each order of t_1 and t_2. With more than two instructional backgrounds and a larger number of discriminating items, the magnitude order of two response times would be generalized to a vector of response times exhibiting different patterns, i.e., permutation of the magnitudes of the elements.

Person Conditional Response Rate (PCRR)

Let me backtrack a little bit and explain why the Weibull family was chosen over the gamma despite the latter's having a longer tradition of usage in response-time models (Rasch, 1960; Restle & Davis, 1962, among others). First, the gamma distributions are indicated when distinct stages are identifiable in the process of solving the tasks, in which case c must be an integer representing the number of stages. Second, the shape parameter c of the Weibull family has the interesting feature of apparently distinguishing between different information-processing skills associated with different instructional backgrounds. This feature is no doubt related to the fact that the magnitude of c (i.e., whether c is greater than, equal

to, or less than 1) determines the nature of the item conditional response-rate function (ICRR), which describes whether perseverance increases the chances of an examinee's responding to an item, responses occur at random times, or a point of diminishing returns is reached early. In other words, it can be said as mentioned in the first section on "Rationale of Weibull Distributions," that c is sensitive to the degree of involvement students show. Two different instructional methods usually require different steps of information-processing skills, thus each method requires a different degree of involvement in solving a given item. For example, some items in Table 1, such as "10+-11 = ?" in the signed-number posttest, yield not only different values of c, but also significantly different mean response-times depending on whether the sequential or number-lines method is used for answering it, as dictated by the examinee's instructional background. Moreover, the convenient ICRR function is readily expressed in closed form for a Weibull distribution, but cannot be so expressed for a gamma distribution because the incomplete gamma function cannot be expressed analytically.

We first present the ICRR function, i.e., the probability that an examinee who has not responded to an item by time t will do so within an infinitesimal time interval thereafter. When item response times follow a Weibull distribution, this function $H_g(t)$ is given by $f_g(t)/[1- F_g(t)]$, where $f_g(t)$ and $F_g(t)$ are expressions (1) and (2) with the parameters subscripted with a g for item g and, in our case, t_0 set equal to 0. Hence

$$(6) \quad Hg(t) = c_g \, t^{c_g - 1} \Big/ u_g^{c_g}$$

From this the transition to PCRR is made in a manner analogous to going from an item characteristic curve to a person characteristic curve, first suggested by Mosier (1936), recently by Weiss in 1973, and discussed in greater detail by Lumsden in 1978. In their studies a plot is made for each individual of the proportions of items of varying difficulty (represented by the horizontal axis) which are passed by that individual. In our case, the ordinate at each point along the horizontal axis representing the mean response time of an item would be the value of $H_g(\tau)$--where τ is the latent response time for the particular person--computed from (6) with the parameter values proper to that item substituted for u_g and c_g. Note that when shape parameter c equals unity for all items, the PCCR curves are identical for all persons. Thus, utilizing the negative exponential distributicn (i.e., a special case of the Weibull distribution functions) for our purpose will not work.

Equation (6) defines a function whose curve characterizes behavior of item g over time in terms of the probability of reaching an answer. The steeper the slope of a curve is, the greater the chance that item g will be solved as time goes by. The steepness of curves is a characteristic attributed to a given item, similar to item discrimination index in latent trait theory. We can define a sort of conditional response rate function on variable τ as follows:

$$(7) \qquad H_{d_g}(\tau) = \frac{c_g}{u_g}\left(\frac{\tau + d_g}{u_g}\right)^{c_g - 1}$$

Similarly, a sort of person conditional response rate function is given as a function on a set of Equations (7), $H_{d_g}(\tau)$, g=1,...,n.

For a fixed person i,

$$(8) \quad G_i(H_{d_g}) = H_{d_g}(\tau_i), \; g = 1, \ldots, n.$$

It should be noted that the τ in Equations (7) and (8) is merely an arbitrary time point and bears no relation to a person's latent response-time (except for coinciding in numerical value). Only in the context of the random variable $\tau + d_g$ does have the sense of latent response time, but to use $\tau + d_g$ as the argument of $H_{d_g}(\cdot)$ would be meaningless because $\tau + d_g$ is approximately the person's observed response time for item g, and it would be a contradiction in terms to speak of the person's responding in the next moment given that he/she has not responded up to the actual time point at which he/she did respond.

The above remarks indicate that the particular approach attempted here for defining PCRR was futile, but <u>not</u> that the concept of PCRR itself is meaningless. An alternative, more justifiable approach might be to transform response time to an approximate normal variable. The transformation we derived by the usual method of obtaining variance-stabilizing transformations was unusable because it was an arcsine transformation whose argument could exceed one.[1] We therefore extended the usual method by taking up to the second term, instead of only the first, in the Taylor series expansion on which the transformation is based. The result was that the transform y is the solution of the following, rather formidable differential equation:

---

[1]This fact was noticed and pointed out to us by Jim Paulson at the conference.

Where

(9) $h(t)(y'')^2 + g(m)y''y' + f(m)y' = C,$

where

$$h(t) = \mu_o^3[\Gamma(1+4/c) - \Gamma^2(1+2/c)]/4 - [\mu_o^3\Gamma(1+3/c)(t-t_o) - 2\mu_o^2\Gamma(1+2/c)(t-t_o)^2 + (t-t_o)^4]$$

$$g(t) = \mu_o^3\Gamma(1+3/c) - 3\mu_o^2\Gamma(1+2/c)(t-t_o) + 2(t-t_o)^3$$

$$f(t) = \mu_o^2\Gamma(1+2/c) - (t-t_o)^2$$

and c is an arbitrary positive constant. (We have not solves this yet but a mathematician colleague assures us that it is soluble!) If we are further willing to assume that τ is normallly distributed (which seems reasonable by virtue of the central limit theorem, since τ is a person's mean response time over an infinite set of items which may be regarded as exhibiting local independence if unidimensionality holds), then y and τ would jointly follow a bivariate normal distribution. Hence, if their correlation, ρ, can be be estimated (roughly analogous to communality estimation in factor analysis), the joint distribution would be uniquely determined. From this and the distribution of τ, the conditional distribution of y given can be determined. All quantities associated with persons having a particular τ-value are computed from this conditional distribution.

ESTIMATION OF THE PARAMETERS

We are now interested in estimating c_g, u_g and τ_I, g=1,...,n, i=1,...,N simultaneously. We have to choose the set of admissible values of these parameters which makes the log-likelihood function, lnL, the

maximum. Unlike the case of dealing with performance scores, response time represents two different cases; one is a group whose members reached the correct answer, while the other group consists of those ending up with wrong answers. Response time in the OK-subgroup means the time needed to attain a given goal using a successful information-processing skill (or skills), but it is not that simple with the NO-subgroup. A brief investigation of error analysis for the NO-subgroup indicates that various kinds of misconceptions at different progressive stages of reaching the correct answer for a given item might have occurred. Therefore, we will consider only the OK-subgroup for our purpose in this paper.

Differentiating the logarithm of Equation (5) by parameters c_g and u_g respectively, and setting the results equal to zero, we obtain the following simultaneous equations.

$$(10) \qquad \frac{\partial \ln L}{\partial c_g} = \sum_i \left[\frac{1}{c_g} + \ln \left(\frac{\tau_i + d_{ig}}{u_g} \right) \left\{ 1 - \left(\frac{\tau_i + d_{ig}}{u_g} \right)^{c_g} \right\} \right] = 0$$

$$(11) \qquad \frac{\partial \ln L}{\partial u_g} = \sum_i \frac{c_g}{u_g} \left[\left(\frac{\tau_i + d_{ig}}{u_g} \right)^{c_g} - 1 \right] = 0$$

$$(12) \qquad \frac{\partial \ln L}{\partial \tau_i} = \sum_g \left[\frac{c_g - 1}{\tau_i + d_{ig}} - \frac{c_g}{u_g} \left(\frac{\tau_i + d_{ig}}{u_g} \right)^{c_g - 1} \right] = 0$$

The maximum likelihood method using the Newton-Raphson iteration procedure provides estimates of the roots of Equations (10) and (11) where, i=1,...,N are substituted by the mean response time of each person over items, g=1,...,n.

Then, the roots of Equation (12) after the newly estimated c_g and u_g values are substituted are sought by the same procedure.

A sufficient but not necessary condition that any of these stationary values u, c, be local maxima is that

$$(13) \qquad \frac{\partial^2 \ln L}{\partial c_g^2} = -\sum_i^{N_g} \left[\frac{1}{c_g^2} + \left\{\ln\left(\frac{\tau_i+d_{ig}}{u_g}\right)\right\}\left(\frac{\tau_i+d_{ig}}{u_g}\right)^{c_g} \ln\left(\frac{\tau_i+d_{ig}}{u_g}\right)\right] < 0$$

$$(14) \qquad \frac{\partial^2 \ln L}{\partial u_g^2} = -\frac{c_g}{u_g}\left[\sum_i^{N_g}\left\{\left(\frac{\tau_i+d_{ig}}{u_g}\right)^{c_g} - 1\right\} + \sum_i^{N_g} c_g \left(\frac{\tau_i+d_{ig}}{u_g}\right)^{c_g}\right] < 0$$

$$(15) \qquad \frac{\partial^2 \ln L}{\partial \tau_i^2} = -\sum_g (c_g - 1)\left[\frac{1}{(\tau_i+d_{ig})^2} + \frac{c_g}{u_g^2}\left(\frac{\tau_i+d_{ig}}{u_g}\right)^{c_g-2}\right] < 0$$

It should be noted that Inequalities (13) and (15) are always negative but Inequality (14) will be negative only when the estimates of u_g are close enough to be the roots of Equation (11). In some earlier stages of iterations, the condition for u_g to yield a local maximum might not be satisfied. Thus, it is important to select appropriate starting values for the estimates of u_gs.

For estimation of τ, we seek solutions of Equation (12) for which Inequality (15) is satisfied. If $c_g = 1$ for all g, g=1,...,n, then Equation (12) becomes

$$\sum_g \left(\frac{1}{u_g}\right) = 0$$

Since scale parameter u_g equals the mean of observed response time when shape parameter c is equal to unity, this equation becomes equivalent to

$$\sum_g \left(\frac{1}{t_g}\right) = 0$$

The reciprocal of observed mean cannot be zero for any item g. Therefore, there should be some g such that $c_g \neq 1$. This implies that the maximum likelihood method does not work for response time models associated with the negative exponential distributions as long as the models are formulated assuming unidimensionality. Moreover, the notion of "person conditional response rate" which is parallel to that of the person's characteristic curve will not be applicable to these models. This is because conditional response rate functions are always parallel to the horizontal axis when the occurrence of a response is a random event and all random events are assumed to be of the same kind, which is the case of negative exponential distributions.

Numerical Example

A computer program for estimating parameters u_g, c_g and τ_i for g=1,...,n, i=1,...,N was written on the PLATO system by Robert Baillie. The parameters in our model were successfully estimated for some sample data, the pretest data from signed-number arithmetic lessons. Unfortunately, the posttest data of signed-number operations described in "Introduction" and elsewhere (see also Appendix 1-a, 1-b, and 1-c) could not provide stable estimates with this computer program because of both small sample size for Group 2 and too few items--only the 12 items that were of most interest to us. When the observed response-time data were fitted to the Weibull distributions before, it was observed that the items testing the same skill in the pretest showed a systematic change with the estimates of u and t_o according to their order of presentation, even though the difficulties of these parallel items did not show any noticeable change.

With this new model, the changes in the slopes of the parallel items have a strong tendency of being monotonically increasing. For example, Items 3 and 45 ask "-3+2=?" and "-7+5=?", respectively. The dotted lines in Figure 7 are CRR functions associated with the observed response-time, while the solid lines are the theoretically derived CRR functions. It is interesting to note that the random variable $\tau_i + d_{ig}$ can be rewritten as $(\tau_i - \bar{t}_{i.}) + t_{ig} = T_{ig}$. If we denote $\tau_i - \bar{t}_{i.} = -\varepsilon_i$, then T_{ig} can be expressed as $t_{ig} - \varepsilon_i$. Hence, the observed response-time t_{ig} becomes the sum of a <u>true-score-like</u> T_{ig} and an "error" ε_i. Therefore, it might be considered that the theoretical CRR functions are defined on <u>a sort</u> of true-score random variable, T_{ig}.

SUMMARY AND DISCUSSION

The customary method for assigning a score to an individual on adaptive tests--or, for that matter, whenever a latent trait model is employed--is to use the estimate θ of the ability (or achievement) parameter. This may be adequate when the only purpose of testing is to calibrate the individual's ability or achievement level. When the further purpose of using θ as the basis for routing the student to a suitable starting point in a lesson series is involved, however, sole reliance on θ can create serious problems. This is because two examinees may have identical response patterns (and hence θ values) and yet differ drastically in the manners in which they arrived at their answers to the items, right or wrong as the case may be. By "the manners" we mean the cognitive processes and information-processing skills that are

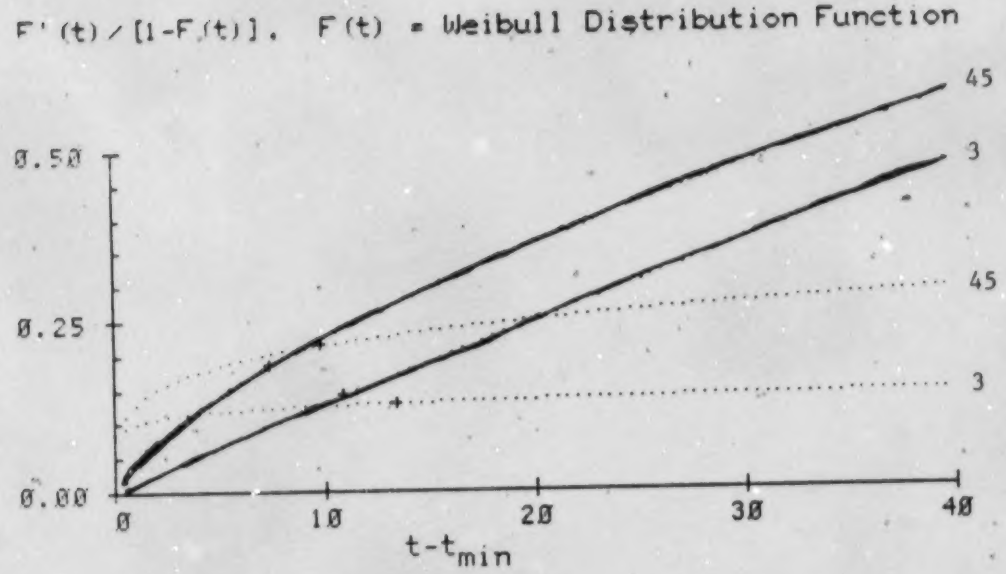


Figure 7 CRR functions defined on the random variable T_{ig}

item	t_0	c	μ_0	$\bar{t}$	
3	5	1.09	8.80	13.517	) observed
45	4	1.22	6.28	9.883	
3	0	1.95	12.35	10.951	) theoretical
45	0	1.68	8.36	7.465	

brought into play. Efficient and effective routing of students to lessons requires this "deeper diagnosis" instead of mere information as to which items they get right or wrong.

Increasing recognition is being given to this fact, as evidenced by the number of studies either directly or indirectly germane to it that have recently been done by cognitive psychologists (e.g., Anderson et al., 1978; Carroll, 1978; Frederiksen, 1978; Greeno, 1977; Groen & Perkum; 1972; Heller & Greeno, 1977; Rose, 1977; Sternberg, 1979). These studies have demonstrated the existence of a variety of cognitive processes which differ from individual to individual.

One clue to the type of cognitive process employed by a student in solving a given problem can come from knowing his/her instructional background. Fortunately, a follow-up study of Tatsuoka and Birenbaum (1979) indicates that the Weibull shape parameter c, obtained by fitting response-time data, is helpful in differentiating among various instructional methods associated with signed-number operations. The Weibull distributions can be mathematically derived from the assumptions that conditional response rate (CRR)--essentially the conditional probability that a person will respond to a given item during the interval [t,t+dt] given that he/she has not responded to the item up to time t--is monotonically increasing, decreasing or constant. The slope of the conditional response rate function for a given item is determined by the magnitude of the shape parameter and the mean of the item response-time. If c is larger than 1, then CRR is a monotonically increasing function. If c = 1, then CRR is constant. Some types of information-processing skills

require a greater amount of involvement in the student's effort in solving a given problem, while others don't require so much to get the answer to the same item. The magnitude of shape parameter c and mean response-time for the former become noticeably larger than those in the latter case. Therefore the slopes of CRR functions differ in steepness to a greater extent. This sensitivity of the Weibull distributions to the procedures associated with different teaching methods is an advantage in dealing with psychological research, as Scheiblechner (1979) states: "The exponential or Weibull distribution is an adequate model for more sorts of psychological data than is commonly assumed if the parameteic structure of the latencies is properly chosen."

First, we assume that for a given set of items there exists a latent variable which affects the time taken by an examinee to answer each of these items. We formulated a model associated with response-time, roughly paralleling latent trait theory, on the strength of the observed fact that the Weibull distribution fits the response-time data for most items. Our main concern in the model is to express the relationship between latent response-time variable and information-processing skills.

An estimation routine of the parameters by the maximum likelihood method was programmed by Robert Baillie on the PLATO system and a numerical example was shown (see Figure 7). The maximum likelihood method is not applicable to estimate Weibull parameters when all shape parameters are supposed to be 1, that is the case of negative exponential distributions. Further research will be necessary in exploring a different parameter estimation procedure, such as the conditional maximum likelihood method.

Information function of item g, $I_g(\tau)$ was integrated numerically and found to be always constant except for $c_g = 1$. However, we relegate its discussion to the next technical report.

The particular approach attempted here for defining item CRR and Person Conditional Response Rate functions resulted in the loss of the attractive feature of being capable of providing mathematical meaning to the curves in terms of τ_i. However, the attractive feature still holds for the variable mentioned in the numerical example, that is, T_{ig}. An alternative approach was outlined, but further research is necessary to make this approach operational.

Appendix 1-a

Kolmogorov-Smirnov Tests for the Total Sample of the Signed-Number Test

item	p	z	N
1)	Ø.42	Ø.88	59
2)	Ø.97	Ø.5Ø	27
3)	Ø.15	1.13	55
4)	Ø.51	Ø.82	47
5)	Ø.Ø1	1.71	68
6)	Ø.98	Ø.48	42
7)	Ø.84	Ø.62	25
8)	Ø.29	Ø.98	46
9)	Ø.75	Ø.68	37
1Ø)	NOT TESTED		
11)			
12)	Ø.13	1.17	87
13)	Ø.99	Ø.43	24
14)	Ø.69	Ø.71	76
15)	Ø.18	1.1Ø	59
16)	Ø.95	Ø.52	25
17)	Ø.Ø9	1.24	59
18)	Ø.15	1.14	6Ø
19)	Ø.23	1.Ø4	6Ø
2Ø)	Ø.87	Ø.6Ø	41
21)	Ø.67	Ø.72	2Ø
22)	Ø.92	Ø.55	37
23)	Ø.88	Ø.59	33
24)			
25)			
26)	Ø.12	1.19	85
27)	Ø.12	1.19	44
28)	Ø.79	Ø.65	71
29)	Ø.Ø1	1.71	57
3Ø)	Ø.59	Ø.77	25
31)	Ø.23	1.Ø4	58
32)	Ø.19	1.Ø8	62

item	p	z	N
33)	Ø.Ø7	1.3Ø	61
34)	Ø.38	Ø.91	41
35)	Ø.18	1.Ø9	19
36)	Ø.39	Ø.9Ø	39
37)	Ø.2Ø	1.Ø7	43
38)			
39)			
4Ø)	Ø.21	1.Ø6	78
41)	Ø.64	Ø.74	47
42)	Ø.63	Ø.75	76
43)	Ø.Ø1	1.63	6Ø
44)	Ø.48	Ø.84	31
45)	Ø.Ø5	1.36	56
46)	Ø.37	Ø.92	52
47)	Ø.49	Ø.83	69
48)	Ø.78	Ø.66	34
49)	Ø.86	Ø.6Ø	23
5Ø)	Ø.34	Ø.94	25
51)	Ø.65	Ø.74	44
52)			
53)			
54)	Ø.Ø3	1.44	86
55)	Ø.89	Ø.58	28
56)	Ø.18	1.Ø9	77
57)			
58)			
59)	Ø.ØØ	1.75	8Ø
6Ø)	Ø.66	Ø.73	21
61)			
62)			
63)	Ø.ØØ	2.22	83
64)	Ø.42	Ø.88	46

Appendix 1-b

Kolmogorov-Smirnov Tests for Group 2 of the Signed-Number Test

item	p	z	N
1)	Ø.98	Ø.47	16
2)	1.ØØ	Ø.26	4
3)	Ø.95	Ø.53	18
4)	Ø.76	Ø.67	13
5)	Ø.42	Ø.88	23
6)	Ø.99	Ø.45	1Ø
7)	1.ØØ	Ø.29	3
8)	1.ØØ	Ø.39	1Ø
9)	Ø.76	Ø.67	8
1Ø)	NOT TESTED		
11)			
12)	Ø.61	Ø.76	26
13)	Ø.86	Ø.61	7
14)	Ø.91	Ø.56	24
15)	Ø.96	Ø.51	16
16)	1.ØØ	Ø.29	3
17)	Ø.98	Ø.46	18
18)	Ø.83	Ø.63	2Ø
19)	Ø.71	Ø.7Ø	19
2Ø)	1.ØØ	Ø.35	8
21)	1.ØØ	Ø.35	2
22)	1.ØØ	Ø.41	6
23)	Ø.99	Ø.44	7
24)			
25)			
26)	Ø.72	Ø.69	26
27)	Ø.94	Ø.54	11
28)	Ø.88	Ø.59	23
29)	Ø.91	Ø.56	16
3Ø)	1.ØØ	Ø.29	3
31)	Ø.9Ø	Ø.57	15
32)	1.ØØ	Ø.3Ø	19

item	p	z	N
33)	Ø.85	Ø.61	2Ø
34)	1.ØØ	Ø.34	7
35)	Ø.ØØ	Ø.ØØ	Ø
36)	Ø.97	Ø.49	1Ø
37)	Ø.87	Ø.59	9
38)			
39)			
4Ø)	Ø.75	Ø.67	26
41)	Ø.77	Ø.66	14
42)	Ø.98	Ø.47	24
43)	Ø.52	Ø.81	16
44)	1.ØØ	Ø.35	2
45)	Ø.78	Ø.66	18
46)	Ø.85	Ø.61	18
47)	Ø.25	1.Ø1	23
48)	Ø.52	Ø.81	9
49)	1.ØØ	Ø.35	2
5Ø)	Ø.98	Ø.47	4
51)	Ø.7Ø	Ø.71	7
52)			
53)			
54)	Ø.38	Ø.91	26
55)	Ø.89	Ø.58	3
56)	Ø.67	Ø.73	25
57)			
58)			
59)	Ø.85	Ø.61	25
6Ø)	1.ØØ	Ø.35	2
61)			
62)			
63)	Ø.88	Ø.59	26
64)	Ø.94	Ø.54	15

Appendix 1-c

Kolmogorov-Smirnov Tests for the

Group other than Group 2 of the Signed-Number Test

item	p	z	N
1)	Ø.45	Ø.86	43
2)	Ø.94	Ø.53	23
3)	Ø.32	Ø.95	38
4)	Ø.82	Ø.63	34
5)	Ø.1Ø	1.22	45
6)	Ø.99	Ø.44	32
7)	Ø.93	Ø.54	22
8)	Ø.89	Ø.58	35
9)	Ø.99	Ø.46	29
1Ø)	NOT TESTED		
11)			
12)	Ø.17	1.11	61
13)	Ø.99	Ø.45	17
14)	Ø.49	Ø.83	52
15)	Ø.64	Ø.74	43
16)	Ø.97	Ø.49	22
17)	Ø.59	Ø.77	4Ø
18)	Ø.31	Ø.97	4Ø
19)	Ø.49	Ø.83	4Ø
2Ø)	Ø.86	Ø.6Ø	33
21)	Ø.43	Ø.87	18
22)	Ø.88	Ø.59	32
23)	Ø.8Ø	Ø.64	26
24)			
25)			
26)	Ø.19	1.Ø9	59
27)	Ø.23	1.Ø4	33
28)	Ø.95	Ø.52	48
29)	Ø.Ø3	1.45	41
3Ø)	Ø.85	Ø.61	22
31)	Ø.4Ø	Ø.89	43
32)	Ø.26	1.Ø1	43
33)	Ø.59	Ø.77	41
34)	Ø.37	Ø.92	34
35)	Ø.18	1.Ø9	19
36)	Ø.36	Ø.92	29
37)	Ø.69	Ø.71	34
38)			
39)			
4Ø)	Ø.16	1.13	52
41)	Ø.93	Ø.54	32
42)	Ø.92	Ø.55	52
43)	Ø.Ø6	1.32	44
44)	Ø.63	Ø.75	28
45)	Ø.14	1.15	38
46)	Ø.59	Ø.77	34
47)	Ø.74	Ø.68	47
48)	Ø.62	Ø.76	25
49)	Ø.9Ø	Ø.57	21
5Ø)	Ø.35	Ø.93	21
51)	Ø.63	Ø.75	37
52)			
53)			
54)	Ø.Ø7	1.28	6Ø
55)	Ø.99	Ø.43	25
56)	Ø.24	1.Ø3	52
57)			
58)			
59)	Ø.Ø5	1.37	55
6Ø)	Ø.72	Ø.69	19
61)			
62)			
63)	Ø.ØØ	2.Ø4	57
64)	Ø.69	Ø.71	31

Appendix 2

$F'(t)/[1-F(t)]$, $F(t)$ = Gamma Distribution Function

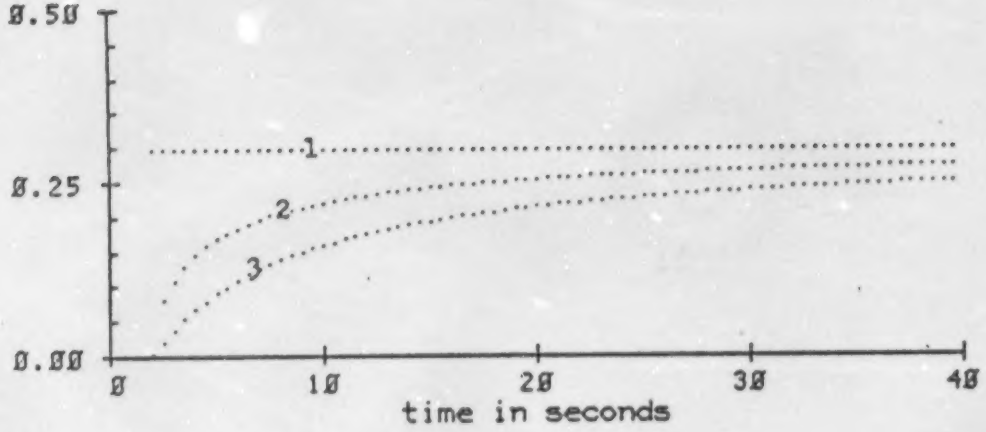


Figure Conditional Response Rate of Three Parameter Gamma Distribution

	$t_Ø$	c	$\mu_Ø$
1	2	1	5
2	2	1.5	5
3	2	2	5

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